

FRACCIONES PARCIALES

El grado mayor de los factores del denominador es 1 y ninguno se repite

Ejemplo:

$$\begin{aligned}\frac{x-4}{x^2+2x-8} &= \frac{x-4}{(x+4)(x-2)} = \frac{A}{x+4} + \frac{B}{x-2} = \frac{A(x-2) + B(x+4)}{(x+4)(x-2)} \\ &= \frac{(A+B)x - 2A + 4B}{(x+4)(x-2)}\end{aligned}$$

$$\left. \begin{array}{l} A + B = 1 \\ -2A + 4B = -4 \end{array} \right\} \Rightarrow A = \frac{4}{3} \quad \wedge \quad B = -\frac{1}{3}$$

$$\frac{x-4}{x^2+2x-8} = \frac{\frac{4}{3}}{x+4} - \frac{\frac{1}{3}}{x-2} = \frac{4}{3(x+4)} - \frac{1}{3(x-2)}$$

El grado mayor de los factores del denominador es 2 y ninguno se repite

Ejemplo:

$$\frac{x+5}{x^3-x^2+x-1} = \frac{x+5}{(x^2+1)(x-1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x-1}$$

$$= \frac{(Ax+B)(x-1) + C(x^2+1)}{(x^2+1)(x-1)} = \frac{(A+C)x^2 + (B-A)x + C - B}{(x^2+1)(x-1)}$$

$$\left. \begin{array}{l} A + C = 0 \\ -A + B = 1 \\ -B + C = 5 \end{array} \right\} \Rightarrow A = -3 \quad \wedge \quad B = -2 \quad \wedge \quad C = 3$$

$$\frac{x+5}{x^3-x^2+x-1} = -\frac{3x+2}{x^2+1} + \frac{3}{x-1}$$

El grado mayor de los factores del denominador es 1 y al menos uno se repite

Ejemplo:

$$\frac{1}{x^2(x-3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-3} = \frac{Ax(x-3) + B(x-3) + Cx^2}{x^2(x-3)}$$

$$= \frac{(A+C)x^2 + (B-3A)x - 3B}{x^2(x-3)}$$

$$\left. \begin{array}{l} A + C = 0 \\ -3A + B = 0 \\ -3B = 1 \end{array} \right\} \Rightarrow A = -\frac{1}{9} \quad \wedge \quad B = -\frac{1}{3} \quad \wedge \quad C = \frac{1}{9}$$

$$\frac{1}{x^2(x-3)} = -\frac{1}{9x} - \frac{1}{3x^2} + \frac{1}{9(x-3)} = -\frac{1}{9x} - \frac{1}{3x^2} + \frac{1}{9(x-3)}$$

El grado mayor de los factores del denominador es 2 y al menos uno se repite

Ejemplo:

$$\frac{x^4 + x^2 + 2}{x(x^2 + 1)^2} = \frac{A}{x} + \frac{Bx + C}{x^2 + 1} + \frac{Dx + E}{(x^2 + 1)^2}$$

$$= \frac{A(x^2 + 1)^2 + (Bx + C)x(x^2 + 1) + (Dx + E)x}{x(x^2 + 1)^2}$$

$$= \frac{A(x^4 + 2x^2 + 1) + B(x^4 + x^2) + C(x^3 + x) + Dx^2 + Ex}{x(x^2 + 1)^2}$$

$$\left. \begin{array}{l} A + B = 1 \\ C = 0 \\ 2A + B + D = 1 \\ C + E = 0 \\ A = 2 \end{array} \right\} \Rightarrow A = 2 \quad \wedge \quad B = -1 \quad \wedge \quad C = E = 0 \quad \wedge \quad D = -2$$

$$\frac{x^4 + x^2 + 2}{x(x^2 + 1)^2} = \frac{2}{x} - \frac{x}{x^2 + 1} - \frac{2x}{(x^2 + 1)^2}$$